Exam C Solutions

Math 13 (Laney, Summer 2004)

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1. *(22 points)* Recall that a roulette wheel has 18 red slots, 18 black slots, and 2 green slots. Consider the following events:

   Event R: the wheel lands on red
   Event B: the wheel lands on black
   Event G: the wheel lands on green

   (a) Compute the probabilities of the following events.

   Answer. $\mathrm{P(R)} = \frac{18}{38} \approx 47.4\%$ and $\mathrm{P(G)} = \frac{2}{38} \approx 5.3\%$.

   (b) Are events R and B complementary? Justify.

   Answer. No, they are not complementary since

   $$\mathrm{P(R)} + \mathrm{P(B)} = \frac{18}{38} + \frac{18}{38} = \frac{36}{38} = 94.7\% \neq 100\%.$$

   (Or you could simply say that there is a third possibility, G.)

   (c) Are events R and G compatible? Explain.

   Answer. No, they are not compatible because the wheel cannot land on both red and green at the same time.

   (d) Compute P(R or B).

   Answer. Since R and B are mutually exclusive,

   $$\mathrm{P(R\ or\ B)} = \mathrm{P(R)} + \mathrm{P(B)} = \frac{18}{38} + \frac{18}{38} = \frac{36}{38} \approx 94.7\%.$$

   (e) Compute P(B & G).

   Answer. B and G are mutually exclusive; that is, they cannot occur at the same time. Hence, P(B & G) = 0.

2. *(15 points)* You toss a biased coin (with 60% probability of heads) 10 times. Let success be defined as heads.

   (a) Is this a binomial procedure? Justify.

   Answer. Yes, because it is a string of IID trials. The trials are independent ("the coin has no memory"), identical ($p = 60\%$ for each trial), and dichotomous (each trials has two possible outcomes: success or failure).

   (b) What is the probability of getting heads 6 times?

   Answer. The probability is

   $$\mathrm{P}(f = 6) = \frac{10!}{6!\,4!}\,(.60)^6(.40)^4 \approx 25.1\%.$$

(c) What is the expected value of the observed success rate?

Answer. Recall that the (observed) success rate is $\hat{p}$. So $\text{Exp}(\hat{p}) = p = 60\%$.

3. *(13 points)* Homer is playing roulette in Las Vegas. He notices that 18 out of the last 22 spins have been black. He tells his daughter Lisa, "Thanks to the law of averages, I can safely bet $100 on red!" What advice should Lisa, who is very smart and knows statistics, give her dad?

Answer. Homer thinks that, since red has come up so often, black is now more likely. But the law of averages does not work by compensating. Lisa should tell her dad that the probability of red coming up in the next spin is still 47.4%, as in every spin of the wheel. She should also remind him of what the law of averages says: Over many, many trials, the observed success rate of an event tends to approach its true probability.

4. *(11 points)* In an ABC News poll conducted June 2–6, 21% of the 1001 adults polled said that a law making voting mandatory (with a small fine for those who don't vote) would be good. (Source: www.pollingreport.com) Write a sentence giving a 95% confidence interval for the percentage of American adults who think such a law would be good.

Answer. Since $\hat{p}$ is close to 20%, we use $80\%/\sqrt{n}$ for E (from the provided table):

$$E \approx \frac{80\%}{\sqrt{1001}} \approx 2.5\%.$$

Thus, we are 95% confident that the percentage of American adults who think such a law would be good is between $21\% - 2.5\% = 18.5\%$ and $21\% + 2.5\% = 23.5\%$.

5. *(15 points)* Consider the following data and events.

	Male	Female	Total
Under 18	6	8	14
18 to 21	14	20	34
Over 21	12	20	32
Total	32	48	80

Event F: picking a female student
Event A: picking a student that's 18 to 21

(a) Compute P(F, given A).

Answer. Out of the 34 people in the 18–21 subgroup, 20 have the characteristic of being female, so P(F, given A) $= \frac{20}{34} \approx 58.8\%$.

(b) Compute P(F, given not A).

Answer. Out of the $80-34 = 46$ people not in the 18–21 subgroup, $48-20 = 28$ have the characteristic of being female, so P(F, given not A) $= \frac{28}{46} \approx 60.9\%$.

(c) Compute P(A, given F).

Answer. Out of the 48 people in the subgroup of females, 20 have the characteristic of being 18–21, so P(A, given F) $= \frac{20}{48} \approx 41.7\%$.

(d) Are the events A and F independent? Justify.

Answer. No, because P(F, given A) $\neq$ P(F, given not A).

6. *(24 points)* Consider your favorite binomial procedure with 200 trials and probability of success 62%. Suppose you want to find the probability that the number of successes is between 128 and 132.

(a) Can you use a normal curve to find the probability? Why, or why not?

Answer. Yes, since the conditions are satisfied:

$$n = 200 > 100, \quad pn = (.62)(200) = 124 > 5, \quad \text{and} \quad qn = (.38)(200) = 76 > 5.$$

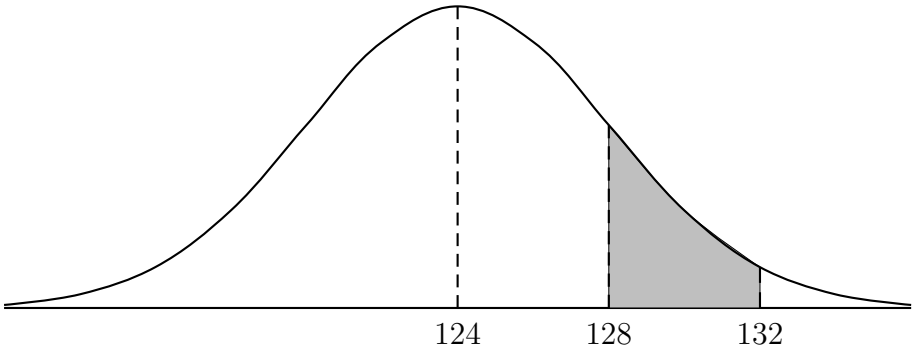
(b) Find the mean and the standard deviation for the number of successes.

Answer. Using the provided formulas,

$$\text{mean} = \text{Exp}(f) = pn = 124 \quad \text{and} \quad \text{SD}(f) = \sqrt{npq} = \sqrt{(200)(.62)(.38)} \approx 6.86.$$

(c) Using the provided standard normal distribution table, find the probability that the number of successes is between 128 and 132. (Show your steps.)

Answer. It really helps to draw a picture.



The z-score of 128 is $\frac{128-124}{6.86} \approx 0.58$, which gives an area of 21.90% (from the standard normal distribution table). The z-score of 132 is $\frac{132-124}{6.86} \approx 1.17$, which gives an area of 37.90%. So the probability we want (i.e., the shaded area) is $37.90\% - 21.90\% = 16\%$.

(d) What is the (exact) probability that the observed success rate is within 1.5 standard deviations of the mean?

Answer. Looking up $z = 1.5$ in the standard normal distribution table, we get the area 43.32%. But that's only for one side, so the probability is

$$2 \cdot 43.32\% = 86.64\%.$$